STATISTICAL ANALYSIS OF EXPERIMENTAL DATA OBTAINED IN WEAR EXPERIENCES OF DENTAL MILLS

Alexandru Saracin[1], Petru Cardei[2], Gheorghe Voicu[1], Ilie Filip[1]
[1]University Politehnica of Bucharest, Romania; [2]National Institute of Research-Development for Machines and Installations Designed to Agriculture and Food Industry, Romania
saracin.alex@gmail.com, petru_cardei@yahoo.com, ghvoicu_2005@yahoo.com, ilie.filip@yahoo.com

Abstract. In this paper we present elementary results of the statistical study by interpolating the experimental data obtained in working experiences with dental mills. The results of the polynomial interpolation of the experimental data leading to polynomial functions that approximate well the dependent parameters of the experimental program are presented. The polynomial interpolation functions can be used to describe the process of wear of the milling cutters only during the experimental working range. Extrapolations do not give satisfactory results. For this reason, there is also a type of interpolation based on controlled functions for large values and null values of the arguments. These interpolation functions, based on an exponential component, are based on physical hypotheses with origins in experiments. The behaviour of these interpolation functions allows the extrapolation of the results. Within this interpolation process, the transition from two independent experimental parameters to one is defined, also reciprocally. Thus, interesting conclusions are drawn regarding the phenomenon that causes wear and which is not the time.

Keywords: dental, mills, tests, statistic, models.

Introduction

The experiences, which data are processed and statistically modelled in this paper, were presented in [1]. Experimental results close to those obtained in [1], for confirmation, can be seen, for example, in [2].

Generally, the phenomenon of wear of surfaces in contact, in movement, is difficult to model by a purely theoretical approach, although not impossible [3; 4]. In the case of the working process of the dental mills, the phenomenon is, at least until the appreciable wear of the milling machine, closer to the cutting process. Empirical or theoretical-empirical approaches to this problem are more common [5-7]. Empirically based mathematical models have been developed since the middle of the twentieth century [8-10].Important advances in the mathematical modeling of the phenomenon of wear of metallic surfaces in contact have been made starting from the process of cutting metals [11-13].After all, the process of dental milling is very similar to a fine cutting process. In [11], four mathematical models are given for the phenomenon of wear of the flanks of metalworking tools: the abrasive model [14] for coated carbide – low carbon steel, the diffusive model [15], for HSS tool C-45, the model of the Taylor equation modified for tool life, [16], and the adhesive-diffusive model [17], coated – carbide low alloy steel.

Complex experimental studies on dental blocks are currently being conducted in advanced research [18]. Interesting experimental results are presented in connection with the milling process of dental millings in [19]. We refer in particular to the fact that pressure values are given in the contact areas, in connection with their wear, even if not necessarily on the head of a dental cutter.

In this paper we present statistical models of tooth milling wear, in [20] we gave a deterministic model, differential for the working process of tooth cutters.

Materials and methods

The working material of the analysis, which results are presented in this paper, is formed by the numerical results of the milling experiences with dental mills, described in [1]. The results of the experiments show the time dependence and the rotation dependence of the milling head, of the geometrical characteristics of the milling head tooth and of the mass of material lost through the wear.The list of these parameters is given in Table 1.

The statistical analysis method used to obtain the results is based on the polynomial interpolation using the technique of the least squares method. Due to the small amount of results, no higher methods of statistical analysis can be addressed, nor can the interpolation polynomials higher than the third degree is obtained by the approach method. However, as it will be seen from the results obtained, the third degree is sufficient to describe the phenomenon of wear of the heads of the dental mills, at least at this stage.

Table 1

Process parameters, name, notations and units of measure

Parameter	Notation	Units
Time	t	seconds, minutes, hours
Mass lost through milling wear	m_{fw}	g
Settlement angle	α	degree
Clearance angle	γ	degree
Sharpening angle	β	degree
Advance	a	µm
Area of the circle at the top	A_{cp}	m^2
Radius of the circle at the top	R_{cp}	m
Rotation speed of the milling head	n	rpm

Results and discussion

The synthesis of the results of the interpolation by the method of the smallest squares for the six parameters measured in experiments is given in Tables 2, 3 and 4. The tables contain the polynomial coefficients of the polynomials by two variables (time and speed of rotation of the head of the dental mill), as well as the average squared error that hierarchies the approximation performances.

Table 2

Coefficients and errors of the interpolation polynomials for the angular parameters, α and γ

Pol. coefficient	α			γ		
	I degree	II degree	III degree	I degree	II degree	III degree
c_{00}	41.267	36.889	35.74	9.314	9.771	9.9
c_{10}	$-5.692\cdot10^{-4}$	$-5.517\cdot10^{-4}$	$-3.071\cdot10^{-4}$	$3.452\cdot10^{-4}$	-3.661·10-4	$6.366\cdot10^{-4}$
c_{01}	$-4.597\cdot10^{-3}$	$1.74\cdot10^{-3}$	0.012	$3.614\cdot10^{-4}$	3.295·10-3	$-2.627\cdot10^{-3}$
c_{11}	0	$-4.39\cdot10^{-7}$	$-7.913\cdot10^{-7}$	0	8.569·10-8	$3.242\cdot10^{-7}$
c_{20}	0	$3.374\cdot10^{-8}$	$2.649\cdot10^{-8}$	0	3.172·10-8	$-1.031\cdot10^{-7}$
c_{02}	0	$-6.156\cdot10^{-7}$	$-1.094\cdot10^{-5}$	0	-8.027·10-7	$2.276\cdot10^{-6}$
c_{21}	0	0	$7.75\cdot10^{-11}$	0	0	$8.455\cdot10^{-12}$
c_{12}	0	0	$-2.152\cdot10^{-10}$	0	0	$-8.718\cdot10^{-11}$
c_{30}	0	0	$-3.775\cdot10^{-12}$	0	0	$4.251\cdot10^{-12}$
c_{03}	0	0	$2.354\cdot10^{-9}$	0	0	$-4.264\cdot10^{-10}$
ε, %	3.141	1.679	0.649	1.735	0.668	0.271

Table 3

Coefficients and errors of the interpolation polynomials for the angular parameter, β and A_{cp}

Pol. coefficient	β			A_{cp}		
	III degree	I degree	II degree	I degree	II degree	III degree
c_{00}	9.9	56.179	62.58	366.251	410.115	441.71
c_{10}	$6.366\cdot10^{-4}$	$1.701\cdot10^{-3}$	$-2.174\cdot10^{-4}$	0.018	-0.053	-0.036
c_{01}	$-2.627\cdot10^{-3}$	$2.477\cdot10^{-3}$	$1.647\cdot10^{-3}$	0.012	0.269	0.325
c_{11}	$3.242\cdot10^{-7}$	0	$4.964\cdot10^{-7}$	0	$3.208\cdot10^{-8}$	$1.478\cdot10^{-4}$
c_{20}	$-1.031\cdot10^{-7}$	0	$7.104\cdot10^{-8}$	0	$4.048\cdot10^{-6}$	$-1.103\cdot10^{-5}$
c_{02}	$2.276\cdot10^{-6}$	0	$-6.623\cdot10^{-7}$	0	$-5.541\cdot10^{-5}$	$-6.072\cdot10^{-4}$
c_{21}	$8.455\cdot10^{-12}$	0	0	0	0	$-1.38\cdot10^{-9}$
c_{12}	$-8.718\cdot10^{-11}$	0	0	0	0	$-2.732\cdot10^{-8}$
c_{30}	$4.251\cdot10^{-12}$	0	0	0	0	$6.17\cdot10^{-10}$
c_{03}	$-4.264\cdot10^{-10}$	0	0	0	0	$1.359\cdot10^{-7}$
ε, %	0.271	1.417	0.565	5.453	4.478	2.675

Polynomial coefficients are double indexed, the first index representing the power in the temporal variable, the second power in rotation. For the calculation of the error, the following formula was used:

$$\varepsilon = 100 \cdot \frac{\sqrt{(y_i - y(x_i))^2}}{\bar{y} \cdot N} \tag{1}$$

where y_i, $i = 1, \ldots, N$ – the experimental data for the dependent variable;
$x_i = t_i\omega_i$ – the data string of the independent variable;
N – the number of experimental data;
$\bar{y}$ – the average of the experimental data for the string of the dependent variable.

The general formula for using Tables 2, 3, and 4, when we want to move to the canonical form of interpolation polynomials, is as follows:

$$p(t,\omega) = \sum_{i=0}^{3}\sum_{j=0}^{3} c_{ij} t^i \omega^j \tag{2}$$

where p – one of the six interpolated parameters (entered first in Tables 2, 3 and 4).

For example:

$$m_{fw}(t,\omega) = -3.733 \cdot 10^{-4} t + 2.056 \cdot 10^{-3}\omega + 6.388 \cdot 10^{-8} t\omega + 4.871 \cdot 10^{-8} t^2 - 1.326 \cdot 10^{-76}\omega^2 - \\ -1.268 \cdot 10^{-12} t^2\omega + 1.385 \cdot 10^{-12} t\omega^2 - 1.774 \cdot 10^{-12} t^3 + 2.241 \cdot 10^{-10}\omega^3 \tag{3}$$

Table 4

Coefficients and errors of the interpolation polynomials for the parameters R_{cp} and m_{fw}

Pol. coefficient	R_{cp}			m_{fw}		
	I degree	II degree	III degree	I degree	II degree	III degree
c_{00}	11.033	11.52	11.865	-0.456	-0.029	0
c_{10}	$2.062 \cdot 10^{-4}$	$-5.426 \cdot 10^{-4}$	$-3.328 \cdot 10^{-4}$	$8.873 \cdot 10^{-5}$	$2.801 \cdot 10^{-5}$	$-3.733 \cdot 10^{-4}$
c_{01}	$1.782 \cdot 10^{-4}$	$2.797 \cdot 10^{-3}$	$3.009 \cdot 10^{-3}$	$4.101 \cdot 10^{-4}$	$1.037 \cdot 10^{-4}$	$2.056 \cdot 10^{-3}$
c_{11}	0	$3.133 \cdot 10^{-9}$	$1.545 \cdot 10^{-6}$	0	$4.891 \cdot 10^{-8}$	$6.388 \cdot 10^{-8}$
c_{20}	0	$4.26 \cdot 10^{-8}$	$-1.192 \cdot 10^{-7}$	0	$-6.492 \cdot 10^{-10}$	$4.871 \cdot 10^{-8}$
c_{02}	0	$-5.696 \cdot 10^{-7}$	$-6.01 \cdot 10^{-6}$	0	$-2.031 \cdot 10^{-8}$	$-1.326 \cdot 10^{-6}$
c_{21}	0	0	$-1.528 \cdot 10^{-11}$	0	0	$-1.268 \cdot e10^{-12}$
c_{12}	0	0	$-2.817 \cdot 10^{-10}$	0	0	$1.385 \cdot 10^{-12}$
c_{30}	0	0	$6.651 \cdot 10^{-12}$	0	0	$-1.774 \cdot 10^{-12}$
c_{03}	0	0	$1.355 \cdot 10^{-9}$	0	0	$2.241 \cdot 10^{-10}$
ε, %	2.375	1.945	1.158	8.3	2.252	1.466

In Fig. 1 the variation is shown of the mass of the milling head, lost through wear during the milling process. The three curves are given by the polynomial equations of the third degree (2), for three values of the rotational speed of the milling head.Also in Fig. 1 there are the points corresponding to the experimental data, based on which the interpolation polynomials were found.

It is observed (Fig. 1), that the mass lost by wear by the milling machine, is generally increasing (except for some very weak minimum points visible at lower rotational speeds). Attention is drawn to avoiding the use of these functions for extrapolation, that is to calculate the loss of mass through wear, for example, at rotational speeds lower than 7000 rpm or higher than 35000 rpm, respectively for time intervals greater than 4 hours (14400 s). The rotation speed of the head of the tooth mill has a greater influence on its wear compared to the temporal parameter, a finding similar to [21].

Fig. 2 shows the surfaces that represent geometrically the interpolation functions of degree I (a portion of the flat surface, blue), II (a portion of the parabolic surface, green), and III (a portion of the cubic surface, red), with two variables: time, in s and speed of rotation of the milling head (rad·s^{-1}). In the same Fig. 2, the experimental data, represented as the centres of diamonds, also appear. The interpolation functions have been exemplified for the case of the variation of the mass lost by the milling head due to wear as functions of time and the rotation speed of the milling head. The same studies and representations can be made for the other five process parameters (Table 1) that characterize the milling head wear.

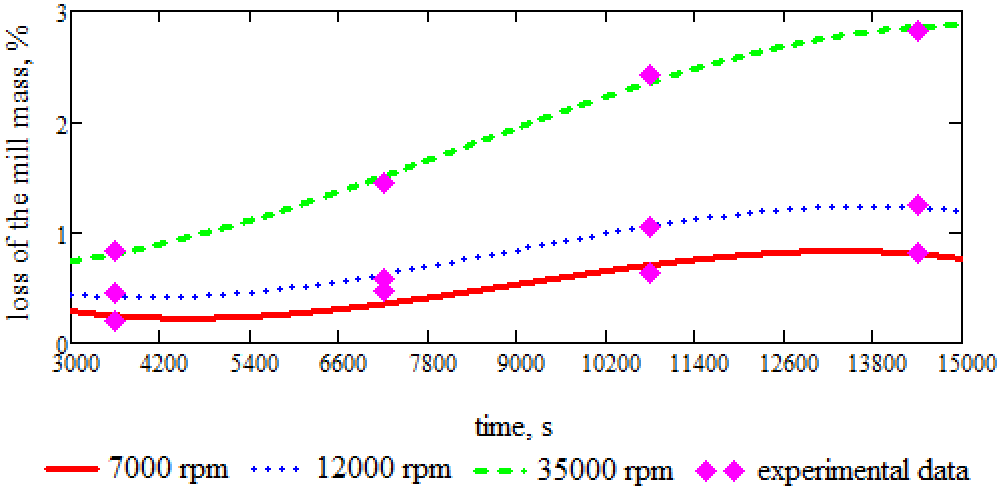

Fig. 1. **Variation with the time, of the material of the milling head lost by wear, for three speed values, the curve is described by the third degree interpolation polynomial**

The interpolation functions for the process parameters that characterize the milling head wear are used for calculating possible extreme points (which could mean optimal reference points), or for estimating any asymptotic behaviour towards large values in time and speed of rotation (which may suggest a quantified limit useful for the replacement of the used dental mill). Any critical or asymptotic points we determine in this way can only be accepted after experimental validation, i.e. after new series of experiences. In addition, the results set out above are limited to the types of dental mills which have been tested and cannot be extrapolated without checks and (in the most favourable case) corrections.

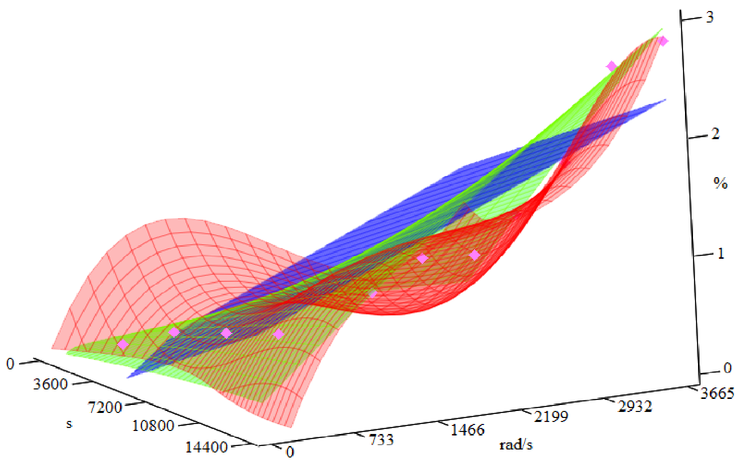

Fig. 2. **Surfaces that represent geometrically the interpolation functions of degree I, II and III, with two variables: time, in s and speed of rotation of the milling heads (rad·s^{-1})**

Polynomial interpolation is a simple case, generally used in numerical data processing, especially when no particular aspects of the studied phenomenon are known. The interpolation polynomials for the dependent parameters of the system, in this case, are as precise as we want, but, from a physical point of view, they do not have, most of the times, a fruitful interpretation. This means that the quantitative accumulations produced by polynomial interpolations do not generally lead to qualitative advances that are for new theoretical explanations and additional applications. In the following, an example of interpolation will be given by a function that has as its source of inspiration, two hypotheses derived from the general experience regarding the processes of wear of the cutting or polishing devices. Also, the hypotheses are in agreement with the observations resulting from the experiments described in [1]. It is considered as a function with the exponential component, which presents an asymptotic behaviour towards infinite plus (which models the stopping of the mass increase after reaching critical wear) and is cancelled at the origin (which is to be expected, in the sense that for zero working time or speed zero rotation, the head of the dental mill does not undergo changes). The expression of this function is elementary and depends on a single argument:

$$\psi(u,c,d) = c\left(1 - e^{-du}\right) \tag{4}$$

where c and d are constant;
u – is the independent parameter (function argument).

The idea of a unique independent variable, starting from the two seemingly independent variables, the time and speed of rotation of the head of the tooth (or the rotation), came from the fact that their product is the angle of rotation of the head of the tooth. Thus:

$$u = t \cdot \omega \tag{5}$$

Now a new hypothesis is introduced for the calculation of the model parameter c. According to the experimental observations, after a complete program of operation the dental mill no longer cuts pieces of the material that is processed. Therefore, we suspect that there is a limit of the rotation angle from which the milling machine no longer works, that is, the amount of material coming from the milling cutter remains approximately constant. This can be interpreted as an asymptotic behaviour towards infinity plus for the function (4). With these hypotheses, the model parameter c is chosen according to the formula:

$$c = \max_{i=1,\dots,N} m_{fwi} \tag{6}$$

where m_{fwi}, $i = 1, \dots, N$ – the experimental data for the mass of material lost from the head of the dental mill by wear.

For the calculation of the second model parameter, d the method of the smallest squares applied to the nonlinear functional is used:

$$\Psi(d) = \sum_{i=1}^{N} \left(\psi(u_i, c, d) - m_{fwi}\right)^2 \tag{7}$$

where

$$u_i = t_i \cdot \omega_i \,. \tag{8}$$

According to (5), and t_i, ω_i, $i = 1, \dots, N$ are the experimental data. The functional (7) being nonlinear, it is numerically minimized and the value of the parameter d is found for the available experimental data set, [1]: $d = 3.5 \cdot 10^{-8}$. According to the experimental data, $c = 2.826$ %. Using these results, the graphical representation of Fig. 3 is made. For comparison, the error in this case is $\varepsilon = 5.306$ %.

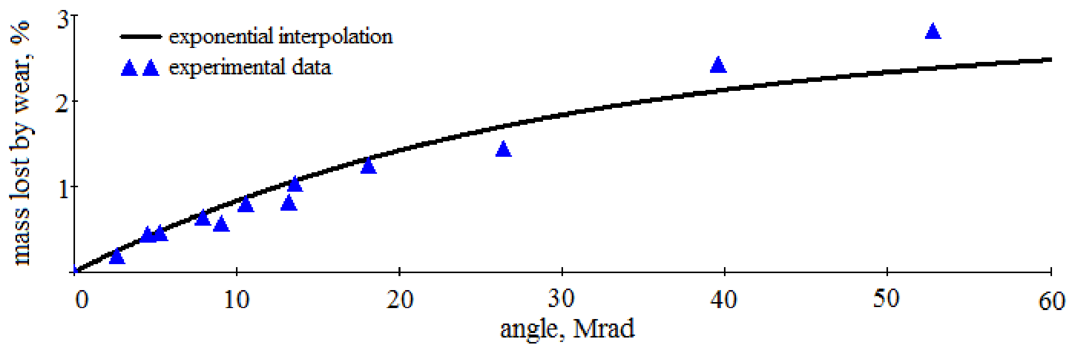

Fig. 3. **Variation of the interpolation function (3) of the mass of material lost through wear by the head of the dental mill and the corresponding experimental data**

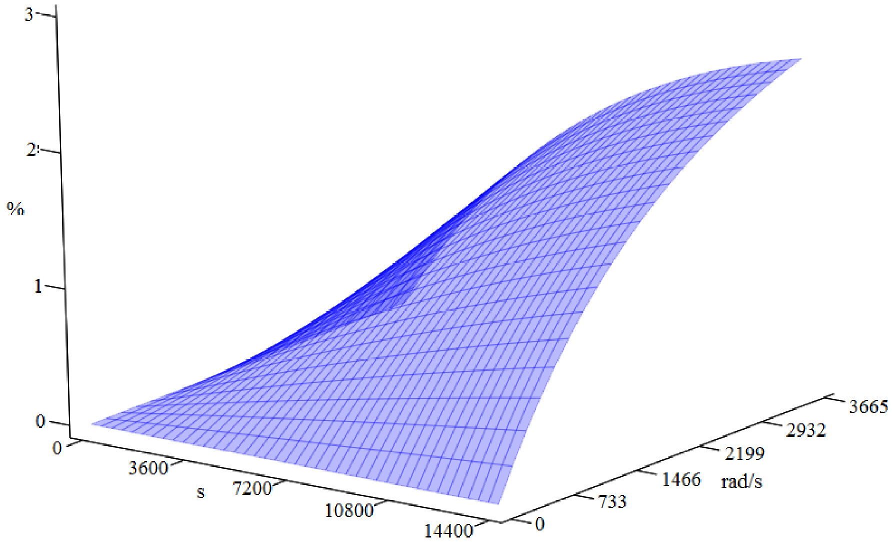

Fig. 4. **Representation of the interpolation function (9) as a function of two variables, which represents the mass of material lost from the dental mill by wear**

By introducing (5) into (4), we obtain the interpolation function of two variables:

$$\psi(t,\omega) = c\left(1 - e^{-dt\omega}\right). \tag{9}$$

Function (9) is represented as a function of two variables in Fig. 4.It is observed that the interpolation function for predicting the wear of the dental cutter head has an important property:

$$\lim_{t\to\infty} \psi(t,\omega) = c > 0 \tag{10}$$

The limit (10) can be used to impose a decommissioning condition. For example, it may be required that when a mass equal to one μ fraction of the limit value c is reached, the dentar mill be declared exhausted. The condition can be written in the following form:

$$e^{-dt\omega} = \mu, \mu \in (0,1), \tag{11}$$

from which a conventional life time of the dental cutter head is obtained:

$$T_\mu = \frac{\ln \frac{1}{1-\mu}}{\omega d}. \tag{12}$$

From (12), it results that the conventional life span thus defined is inversely proportional to the angular velocity of the dental mill head.

In [8], the characterization of the milling wear is made by decreasing the diameter, characterization according to the one given in this article in terms of weight loss through wear.

Conclusions

The main conclusions of this study are as follows:

1. The statistical processing of the experimental data by polynomial interpolation produces functions that represent the variation of the parameters characteristic of the wear process, depending on the varied parameters in experimentation according to the experimental program. These functions are used to determine some characteristic points: extremes, limit values of wear of the head of the dental mill, for example. These functions cannot be used for extrapolation.
2. The interpolation by functions with controlled behaviour at the infinity, and in origin, leads to functions with very likely correct significance in the case of extrapolation of results (which can be proved by experimental validation). The example in which we used the function with the exponential component demonstrates this statement.

3. The two independent variables of the experimentation program can be merged into one, which has the meaning of "angular distance". This shows that time is not the one that produces the wear, but the actual action: the accumulation of an angular distance, which translates into lengths of variable radius on which the cutting and friction leading to wear occur.
4. More extensive experiments are recommended, in which the force of pressing the dental mill on the milled material is varied, so that a wear forecast function with asymptotic variation to be built dependent by the pressing force, which will highlight the influence of this important regime of the regime, together with the other parameters mentioned above.

Acknowledgements

This work has been funded by the European Social Fund from the Sectoral Operational Programme Human Capital 2014-2020, through the Financial Agreement with the title "*Scholarships for entrepreneurial education among doctoral students and postdoctoral researchers* (*Be Entreprenor!*)", Contract No. 51680/09.07.2019 – SMIS code: 124539.

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